Millthorpe School
Mathematics Department

Year 11 Unit 1

Vectors

Prior knowledge: Year 10 unit 3 graphs
Year 9 unit 4 algebraic expressions

Leads onto: Year 11 exams
Year 12 mechanics

What do I need to be able to do?

- Use and understand different representations for vectors
- Complete vector arithmetic - adding, subtracting and scalar multiplication
- Simplify vector expressions
- Represent vectors geometrically
- Use properties of 2D shapes to solve geometric problems
- Use ratio to divide vector paths
- *Produce geometrical proofs of whether points are collinear, and vectors are parallel*

Keywords/formula

Vector: A quantity with both size and direction

Magnitude: the size of a vector, geometrically this is the length of the line

Scalar: a 'normal' number that does not have direction eg 3**c**, 3 is a scalar quantity, **c** is a vector quantity

Resultant vector: the end result from two or more vectors being added together

Equal: vectors are equal if they have the same length and direction

Parallel: vectors are parallel if one is a scalar multiple of the other

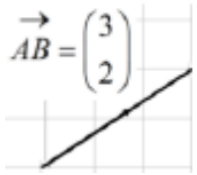
Collinear: Vectors that are parallel and share a common point

Vector algebra

Vectors can be **represented** in several different ways

$$\mathbf{a} \quad \text{or} \quad \overrightarrow{AB} \quad \text{or} \quad \begin{pmatrix}1\\3\end{pmatrix}$$

a represents a path, joining the point **A** to the point **B**



Column vectors represent a translation:
the top number is left/right, the bottom number is up/down

$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up'

$\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'

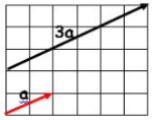
Column vectors can be added and subtracted, or multiplied by a **scalar** value

$= 3\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}4\\-1\end{pmatrix}$

$= \begin{pmatrix}6\\3\end{pmatrix} + \begin{pmatrix}8\\-2\end{pmatrix}$

$= \begin{pmatrix}14\\1\end{pmatrix}$

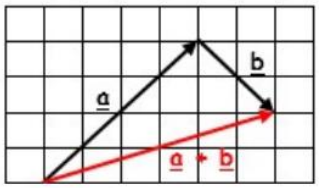
Vectors are **parallel** if one is a scalar multiple of the other
Eg: 2**a** + **b** and 6**a** + 3**b** are parallel

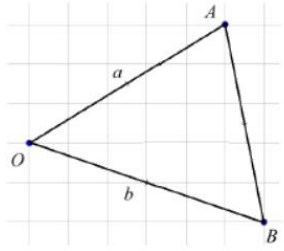


Vector expressions can be **simplified** as algebra
Eg: 2**a** + 4(**a**-**b**) = 6**a** - 4**b**

Vector geometry

Vectors describe paths, and these paths can be combined by following the arrows. You can only move along a path you have a label for.





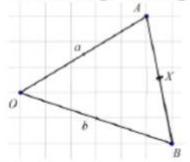
$\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$

$\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$

Example 1: X is the midpoint of AB. Find $\overrightarrow{OX}$
Answer: Draw X on the original diagram



Now build up a journey.

You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$.

This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$.

This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

Vector geometry will use the properties of shapes including parallel sides (rectangles, parallelograms, regular hexagons) and sides of equal lengths (rhombuses, squares, regular polygons

Note: the vectors **a** and **-a** are of equal length and parallel but have arrows in opposite directions